Theory of Relativity
Classical Relativity

The truck is moving toward the Left at 40 mi/hour. Both boys are in the truck and (neglecting wind) they might think they are stationary. If the boy with the blue shirt throws a ball at 10 mi/hour toward the boy with the red shirt, both boys would say the ball is moving at 10 mi/hour.

$$Vt = V1 + V2$$

Theory of Relativity
Classical Relativity

$Vt = V1 + V2$

However, another person standing on the ground would say the ball was moving at 50 mi/hour.
If the boy in the red shirt threw it toward the boy with the blue shirt at 10 mi/hour, again both boys would say it was moving at 10 mi/hour, but a person on the ground would say it was moving 30 mi/hour.
That should make sense. It is called classical relativity.

Theory of Relativity
Classical Relativity

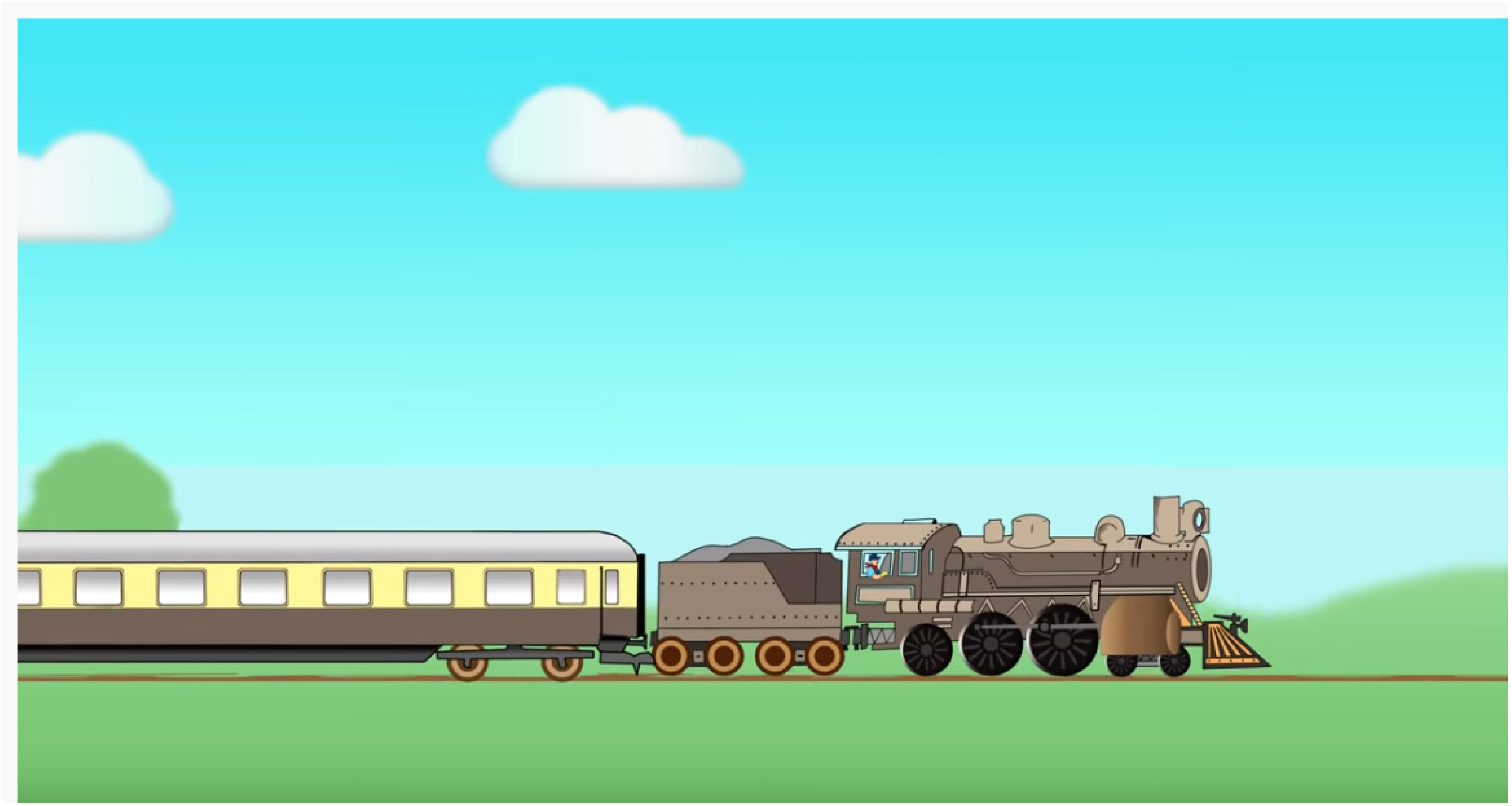
Vt = V1 + V2

It works so long as speeds are much less than the speed of light.
In other words, it works for anything you or I are likely to experience.
However, when speeds approach the speed of light, strange things happen.

Theory of Relativity
Classical Relativity

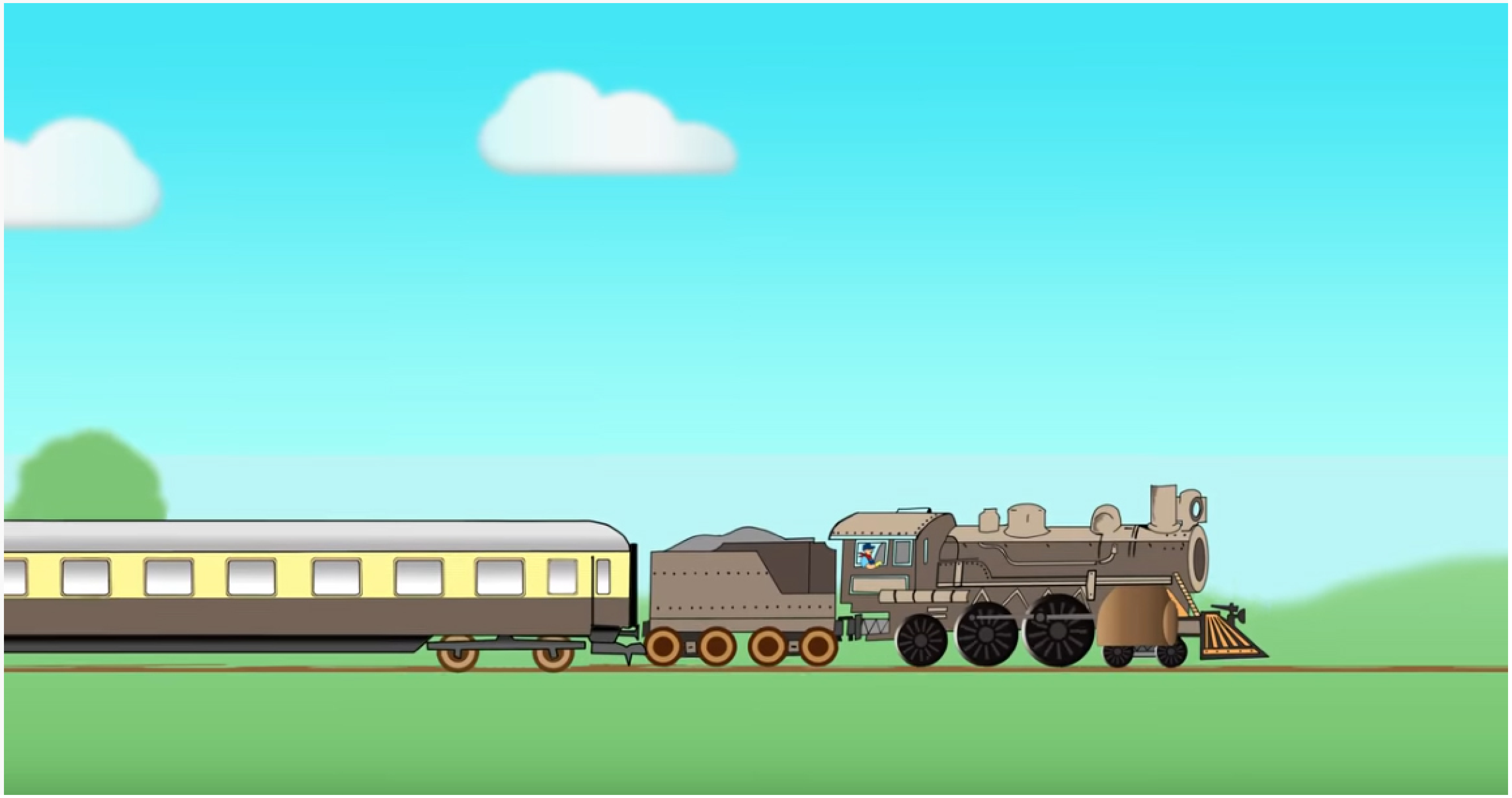
That method of adding velocities, which seems like common sense, comes from the branch of physics called mechanics.

However, equations in another branch of physics called electromagnetism predict something else. Maxwell's equations predict that the speed of light will always be the same value, no matter how the source or observer are moving.



First, the speed of light is 299,792,458 m/s. We'll call that c just to make things easier.

If this train were **standing still** and turned its headlight on, the engineer on the train would measure the speed of the light to be c. And someone on the ground would also measure the speed of the light to be c. So far, so good.



But now say the train is moving toward the right at the speed c/2 and turns on the headlight.

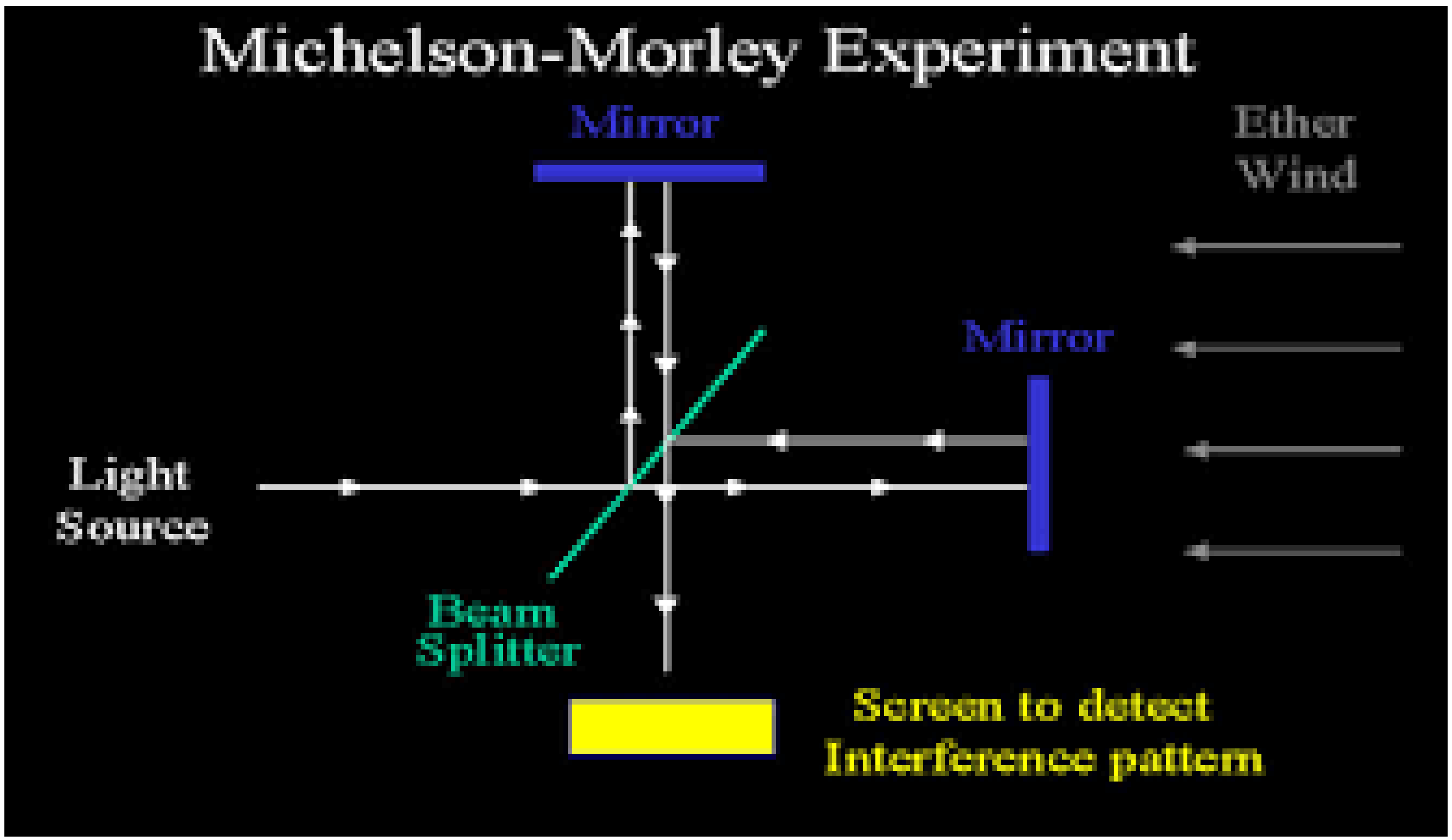
Now the engineer will still see the light traveling at speed c.

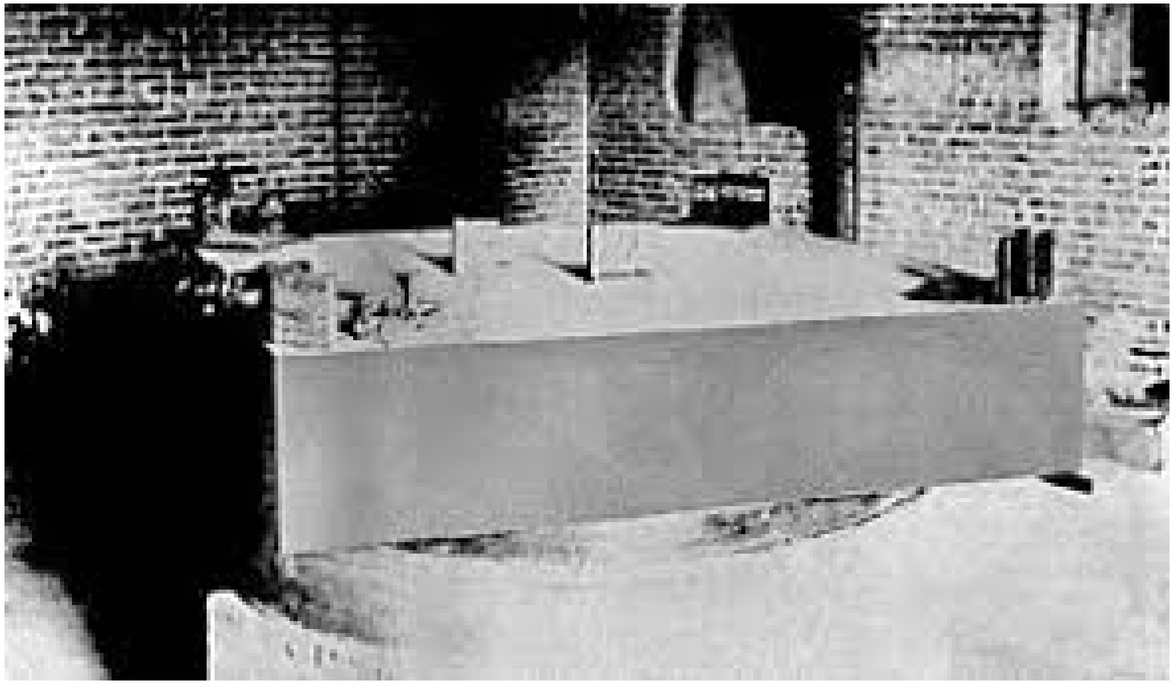
And, a person on the ground will also see the light traveling at speed c.

If you think about that, it does not seem to make sense. However, it is a fact, proven true by many experiments. Read on!

Link to video: https://www.youtube.com/watch?v=7qJoRNseyLQ

A very famous experiment conducted by Michelson and Morley used light interference to prove that the speed of light could be influenced by motion. For example the motion of the earth through space.





Their experiment was on a marble slab (to prevent vibration) which floated on mercury, so they could change its orientation in space. It was an exquisitely sensitive experiment designed to prove that the speed of light could be influenced by motion through space.
And the result was:

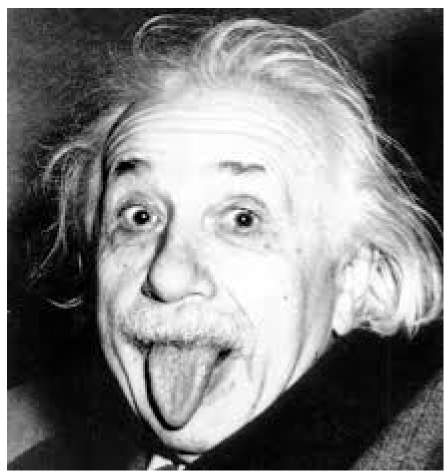

FAILURE

It was a spectacular failure!

No matter how hard they tried, light always traveled at the same speed, c.

Some new theory was needed to explain these experimental observations.

If an experiment's results disagree with theory, then the theory needs to be changed.

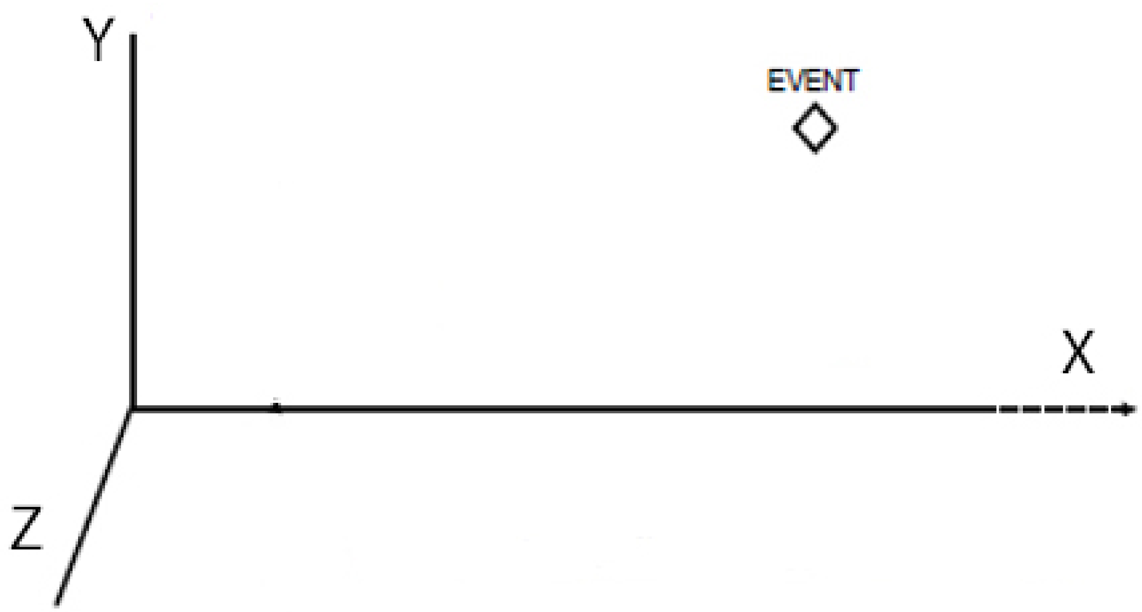


So, Albert Einstein decided he wanted to resolve the apparent contradiction between mechanics electromagnetism.

He started with two postulates.
(A postulate, or axiom, is a statement assumed to be true without proof.)

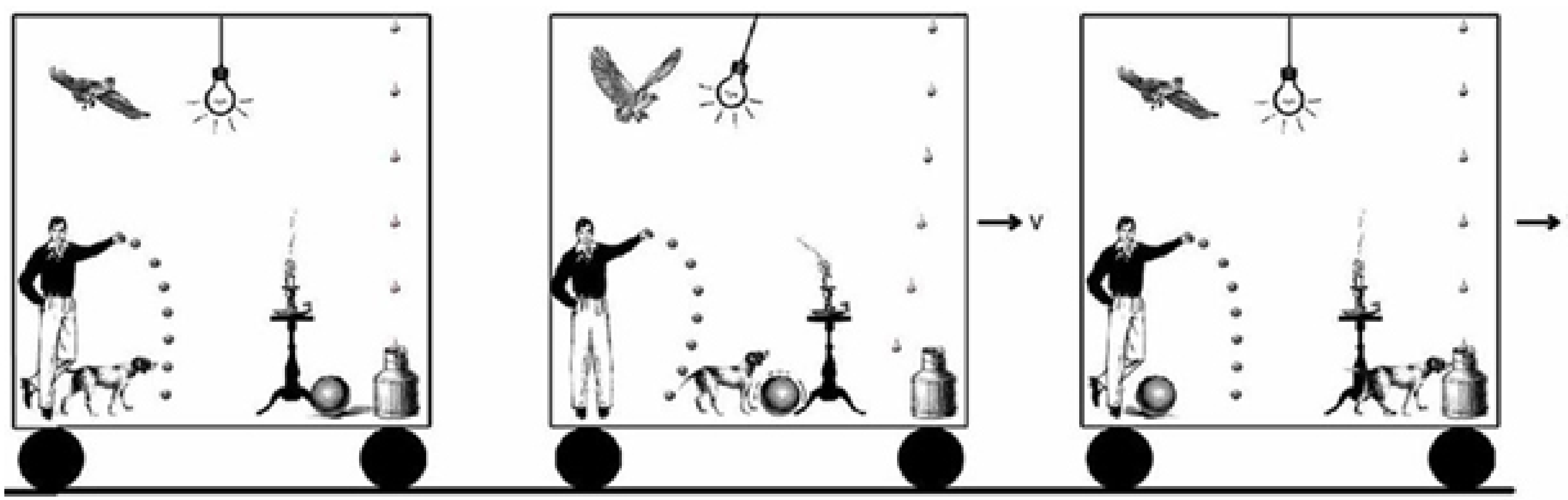
First, let's introduce vocabulary:

Reference Frame: A set of coordinates relative to which measurements are made.



And some more vocabulary:
Inertial Reference Frame:
A reference frame in which the law of inertia holds.
It could be “at rest” or moving at constant velocity with respect to “at rest”.

The equations of motion will be valid in any reference frame that is at rest (A) *or* moving uniformly at a constant velocity (C):



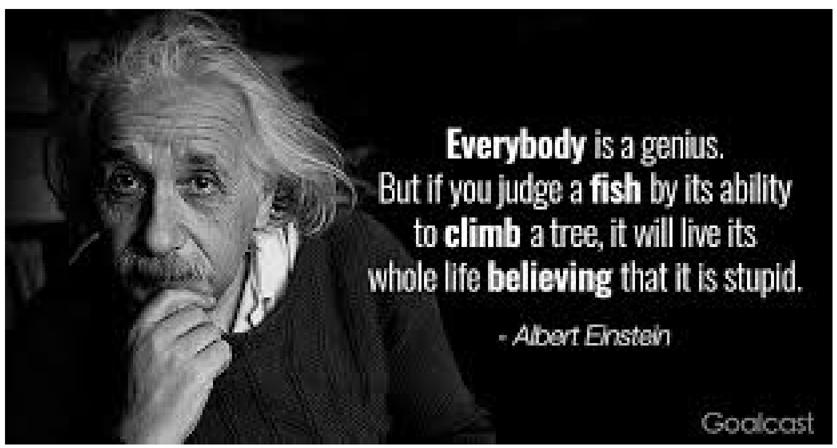
A. Carriage standing still B. carriage accelerating C. carriage moving uniformly

Special Relativity (no acceleration)

Postulates:

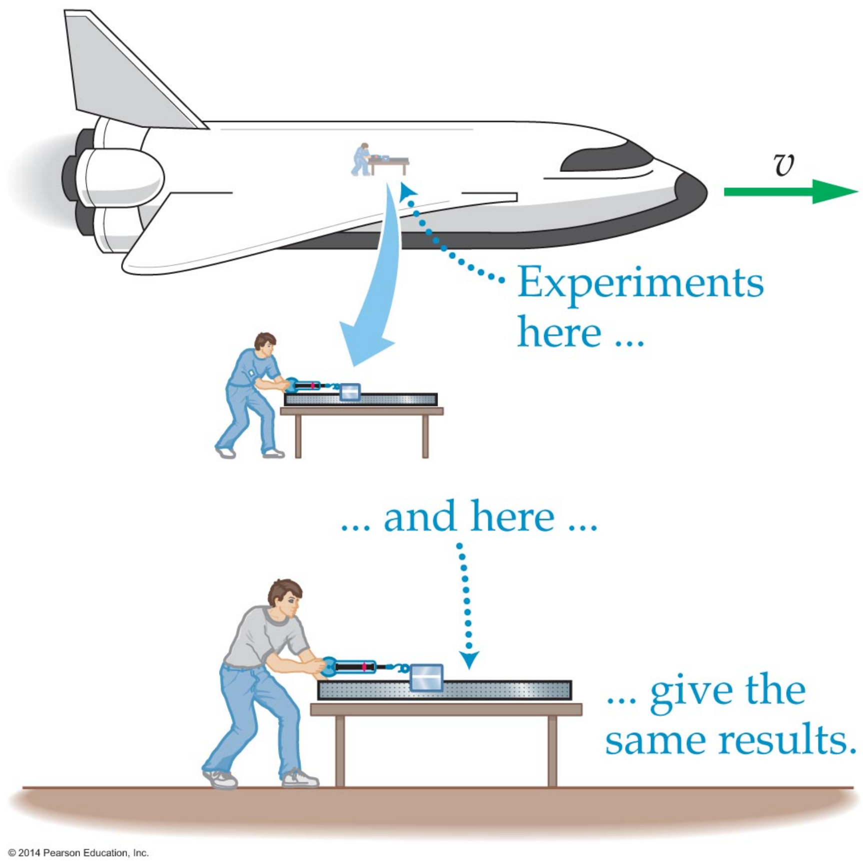
1. The laws of physics are the same in all inertial reference frames.

2. The speed of light in a vacuum is the same for all inertial observers.

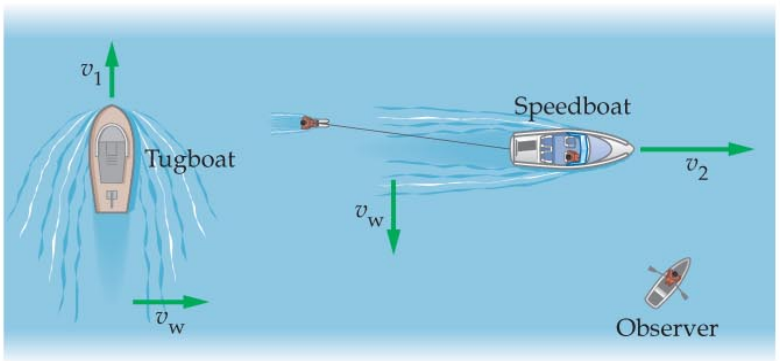


The first postulate seems obvious!

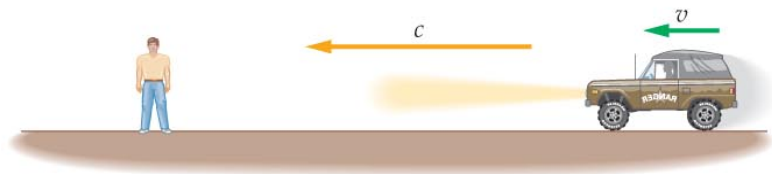
(Experimental results should be the same, no matter where they are done.)



But the second does not!



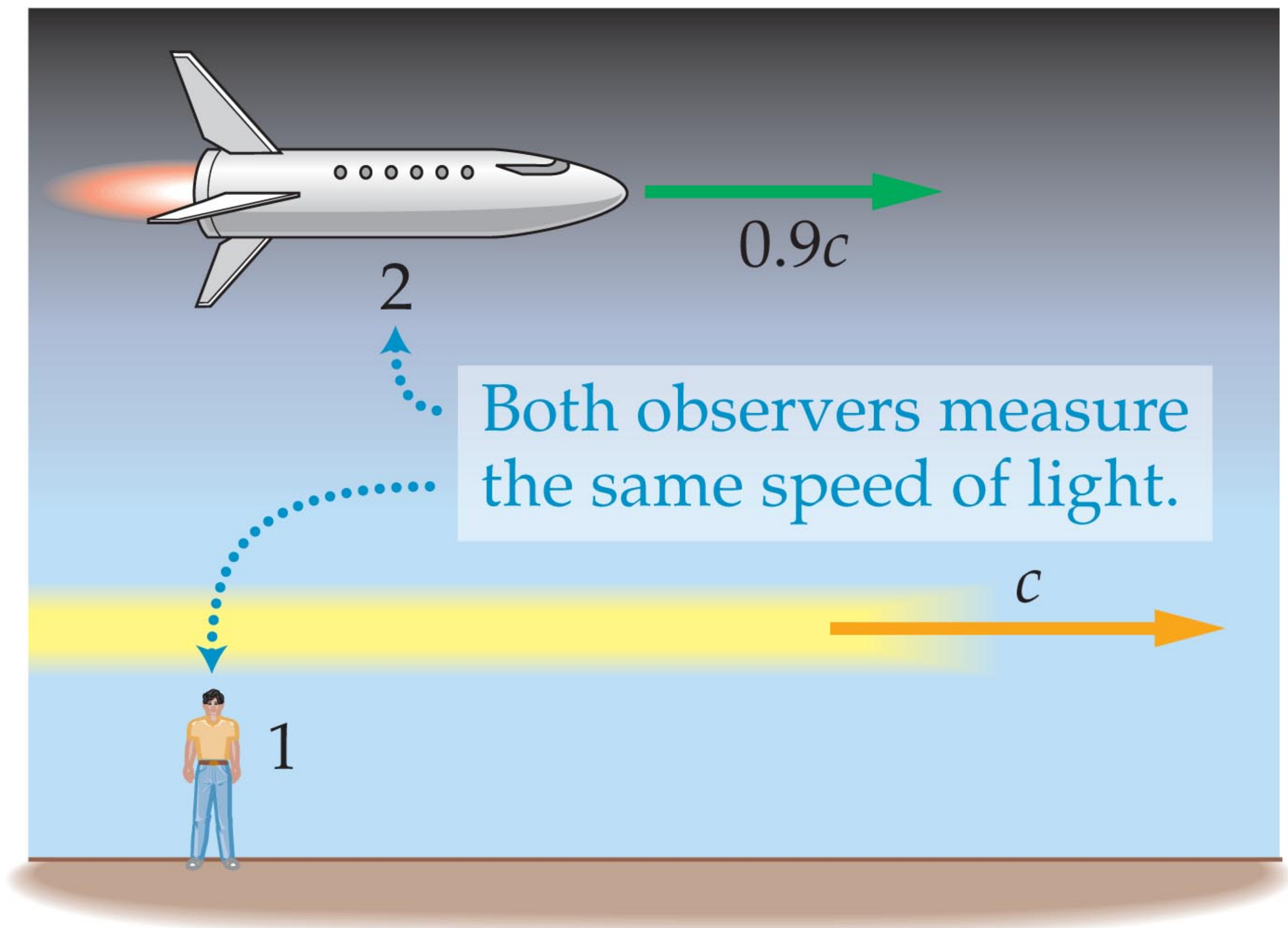
(a) Speed of water waves independent of speed of source

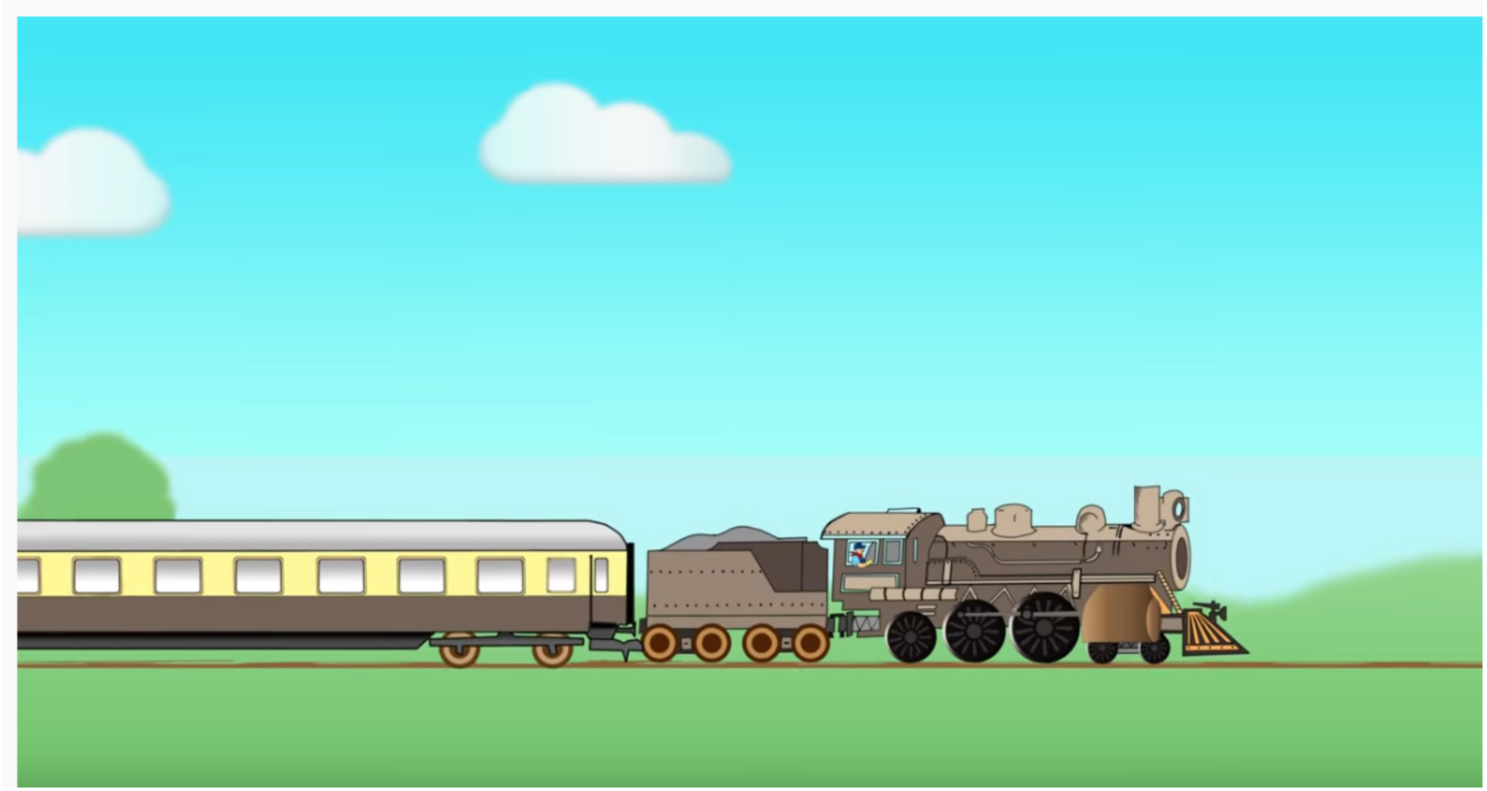


(b) Speed of light waves independent of speed of source

Again, the second does not.

The pilot of the rocket traveling at 0.9C turns on a light and he AND the person on the ground measure the SAME speed for the light, c.
It does not seem logical, but it HAS been experimentally shown to be true.





So, the train is moving toward the right at the speed c/2 and turns on the headlight.

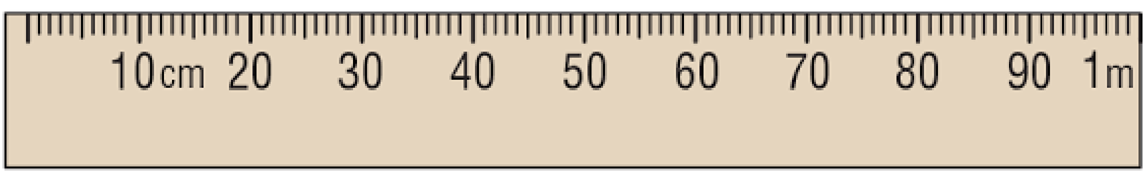
Now the engineer will still see the light traveling at speed c.

And a person on the ground will also see the light traveling at speed c.

Since that seems to go against our common sense**, we need to develop better common sense**.

So, if the postulates are true, we need to rethink a lot of stuff.
For example, TIME and DISTANCE.

We usually assume (as Isaac Newton did,) that time and distance are universal and unchanging.
However since speed = distance/time, and since the speed of light is always constant, then we have to rethink distance and time.

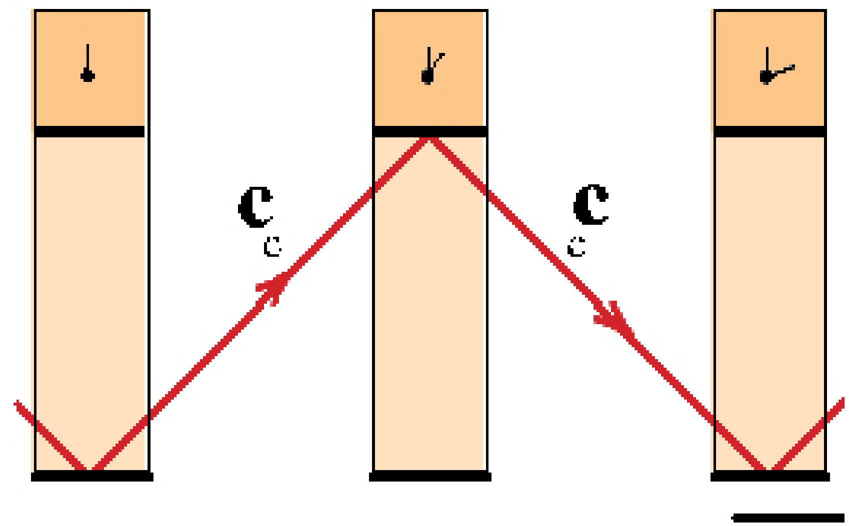


Watch this movie about a light clock
https://www.youtube.com/watch?v=ERgwVm9qWKA&t=36s
(local clip of interest)
http://www.ionaphysics.org/Videos/TimeAndTwinsClocks.mp4

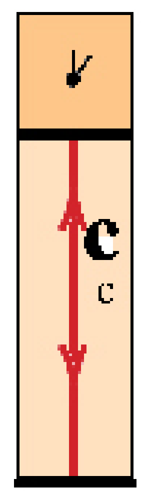
OR:
Consider a beam of light being reflected between two mirrors, separated vertically by a small distance. This clock is stationary.
There is an identical clock moving toward the right at a speed v.

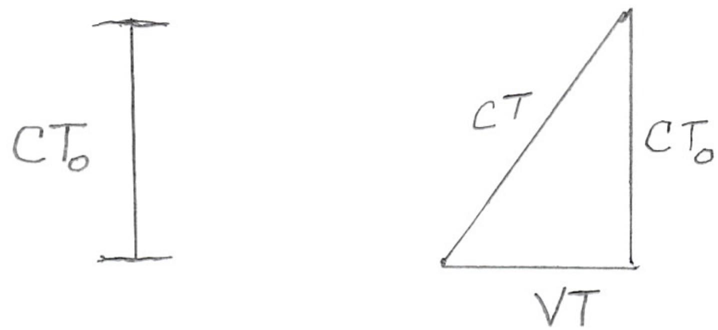
Let's do some algebra and see if we can derive a relationship between the times seen on two clocks by the stationary observer..

The moving observer’s light takes this
longer path



The “stationary” observer’s light
takes this shorter path





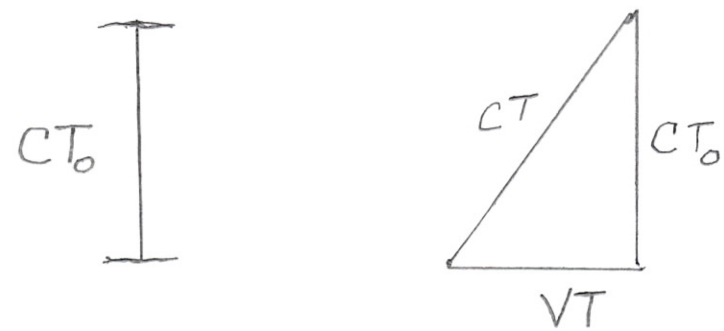
The "Stationary" observer's light clock is on the left. The pulse of light goes from the bottom to the top in time To, covering a distance (d=vt) of cTo.

From the viewpoint of the stationary observer,

the moving clock (shown on the right) is moving toward the right at speed v.

So it covers a horizontal distance vT and the light beam covers a distance cT.

Derivation of Time Dilation



Variables:
To = Time in the rest frame
T = Time in the moving frame (as observed by the rest observer)
c = velocity of light in all frames

Using the Pythagorean Theorem:
(c T) ^2 = (VT)^2 + (cTo) ^2 Complete the squares
C^2 T^2 = V^2 T^2 + c^2 To^2 Collect terms containing T on the left
C^2 T^2 – V^2 T^2 = c^2 To^2 Factor out the T
(c^2-V^2) T^2 = c^2 To^2 Divide both sides by c^2-V^2
T^2 = c^2 To^2/(c^2 - V^2) on right divide top and bottom by c^2
T^2 = To^2/(1-V^2/c^2) Take the square root of both sides
T = To/$\sqrt{1-\frac{V\text{^}2}{C\text{^}2}}$

$$T = To/\sqrt{1 - \frac{V\hat{}2}{C\hat{}2}}$$

As you can see the person who is stationary will observe the moving clock to be ticking slowly, compared to his own clock.

Now let's try a problem:
A plane capable of flying at the speed of sound moves past you, stationary on the Earth. Your clock advanced 3.0000 seconds. How much did the moving clock advance?

$$T = To/\sqrt{1 - \frac{V\hat{}2}{C\hat{}2}}$$

V/c = (331/299792458) = 1 x10^-6
V^2/c^2 = 1x10^-12 which is so close to zero that we can forget it.
Therefore, at least to 11 significant figures, there is no measurable difference.

Well here is another problem:
If 3.0000 seconds went by on your clock, but the plane was a rocket and moving at c/10, would you be able to notice the difference?

T = To/ $\sqrt{1 - \frac{V^2}{C^2}}$

v/c = 0.1
V^2/c^2 = .01

T = 3.0000 s / $\sqrt{1 - .01}$
T = 3.0000 s / $\sqrt{.99}$
T = 3.01511 seconds.

So, approximately .02 seconds longer.

Let's try one more:
If 3.0000 seconds went by on your clock, but the plane was a rocket and moving at 0.9 c, would you be able to notice the difference?

T = To/$\sqrt{1 - \frac{V^2}{C^2}}$

v/c = 0.9
V^2/c^2 = 0.81

T = 3.0000 s / $\sqrt{1 - .81}$
T = 3.0000 s /$\sqrt{.19}$
T = 6.88 seconds.

You certainly would notice that!

You might think this is an interesting theory, but could not really work. Here is an old movie which shows a terrific experiment which actually demonstrated time dilation.

https://www.youtube.com/watch?v=tbsdrHlLfVQ

https://youtu.be/tbsdrHlLfVQ

https://www.ionaphysics.org/Videos/MuonLife.mp4

Time Dilation

$$T = \frac{To}{\sqrt{1 - \frac{V^2}{C^2}}}$$

Length Contraction

$$L = Lo \sqrt{1 - \frac{V^2}{C^2}}$$

Position

$$X = \frac{X - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Lets say that I have a watch and you have a watch. They both work perfectly, and are synchronized. If you go moving past me at any speed, we expect our watches will continue to be synchronized, regardless of the motion.

However, Einstein showed that your watch would run slower than mine and the formula relating them would be

$$T = \frac{To}{\sqrt{1-\frac{V^2}{C^2}}}$$

Summary of equations derived from Einstein's two postulates

Position

$$X' = \frac{X - vt}{\sqrt{1 - \frac{V\hat{}2}{C\hat{}2}}}$$

Adding Velocities
Common sense: Vt = V1+V2

Relativity: Vt = $\frac{v1+v2}{1+\frac{v1*v2}{c\hat{}2}}$

Length Contraction

$$L = Lo\sqrt{1 - (v^2/c\hat{}2)}$$

Time Dilation

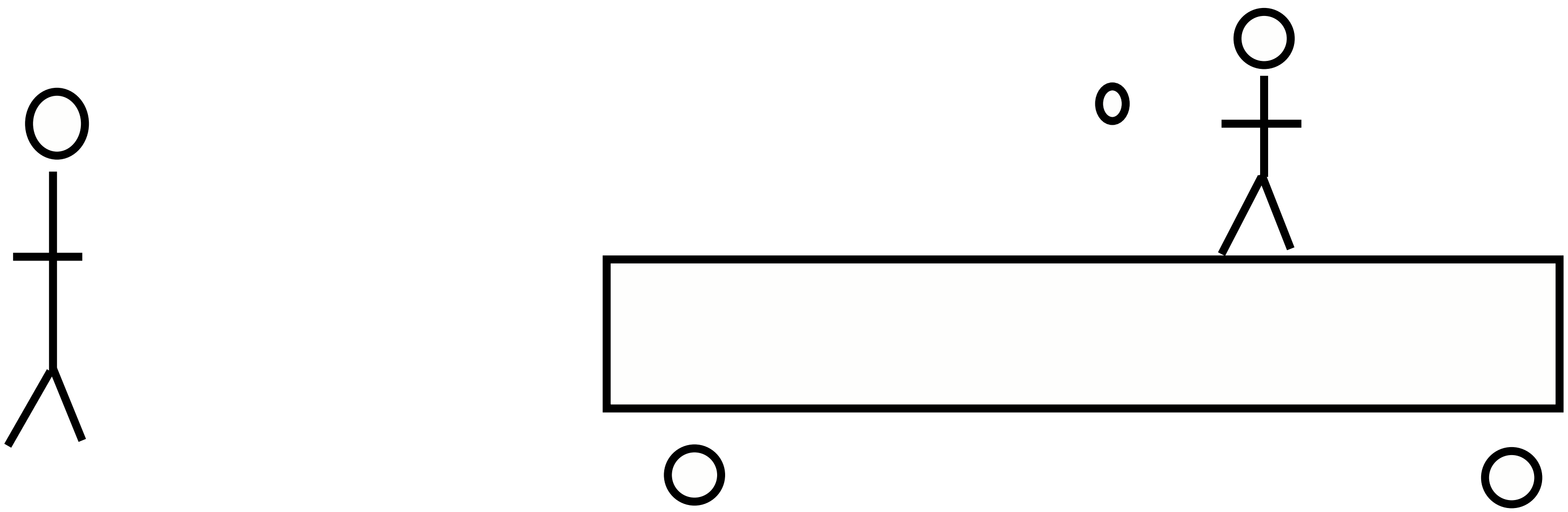
$$T = \frac{To}{\sqrt{1 - \frac{V\hat{}2}{C\hat{}2}}}$$

Mass-Energy
E = mc2

END - SPECIAL RELATIVITY

Worksheets for problems follow

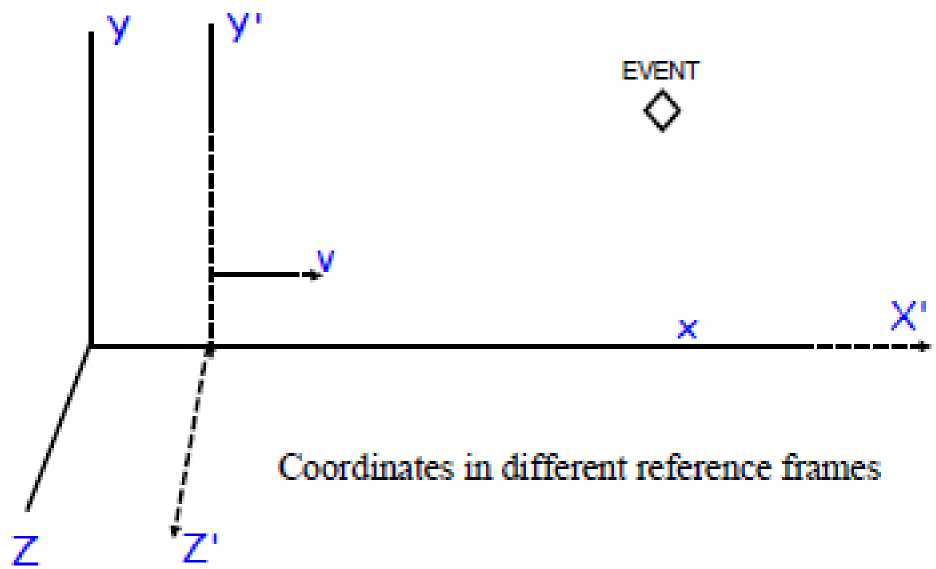
Adding speeds.



$$T = \frac{To}{\sqrt{1 - \frac{V^2}{C^2}}}$$

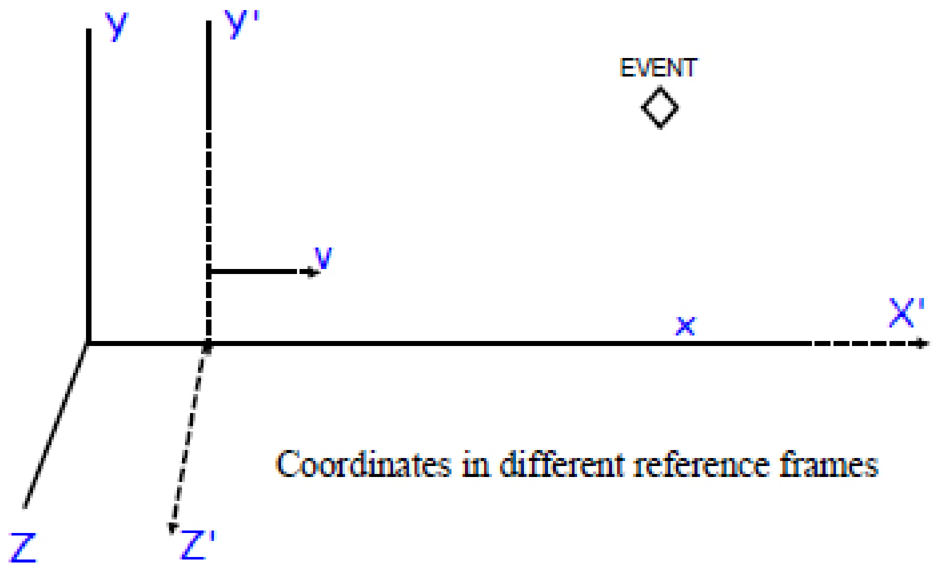
$$L = Lo\sqrt{1 - \frac{V^2}{C^2}}$$

Two observers, one in stationary frame x,y,z
the other in X',Y',Z' moving at constant veloctiy v

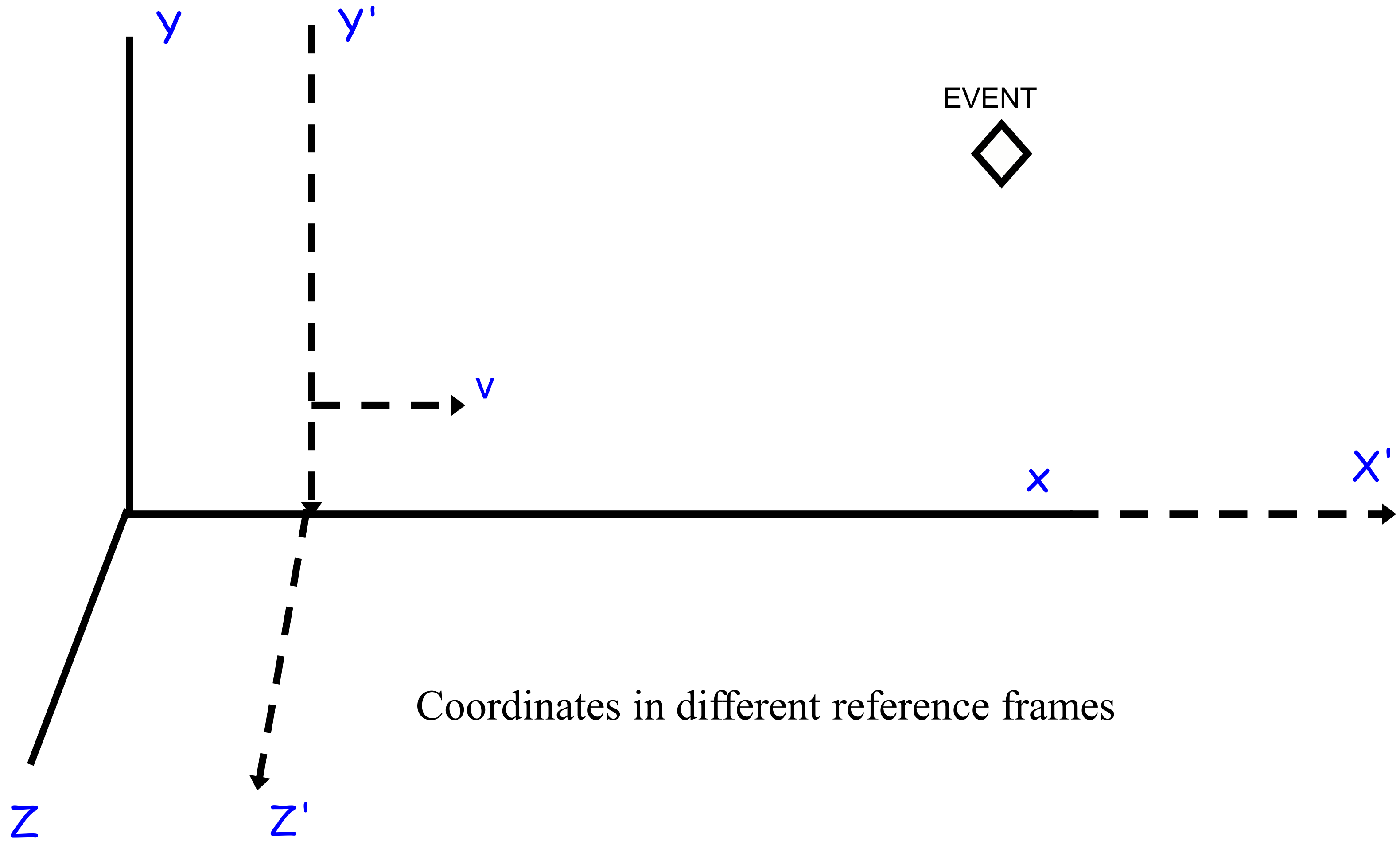


Coordinates in different reference frames

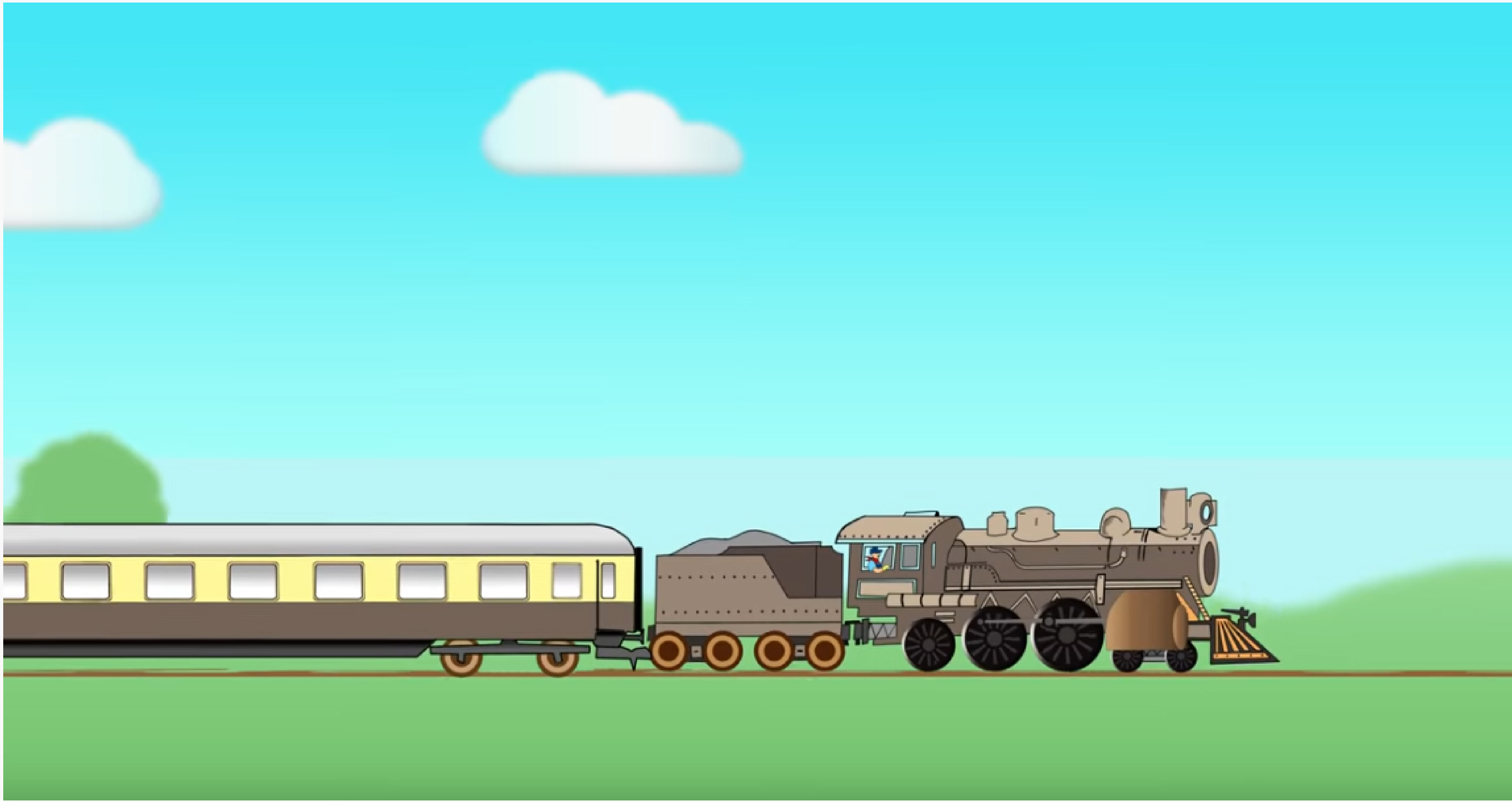
Two observers, one in stationary frame x,y,z
the other in X',Y',Z' moving at constant veloctiy v



Coordinates in different reference frames



Coordinates in different reference frames



Position

X' = $\frac{X-vt}{\sqrt{1-\frac{V^2}{C^2}}}$

E = mc2

Adding Velocities

Common sense: Vt = V1+V2

Relativity: Vt = $\frac{v1+v2}{1+\frac{v1*v2}{c^2}}$

Length Contraction

$$L = Lo\sqrt{1 = v^2/c^2}$$

Time Dilation

$$T = \frac{To}{\sqrt{1-\frac{V^2}{C^2}}}$$

	v/c	v^2/c^2	L if Lo=1	t if To = 1	L*T
Speed of sound	0.000001	1E-12	1	1	1
	0.1	0.01	0.994987437	1.005037815	1
	0.5	0.25	0.866025404	1.154700538	1
	0.9	0.81	0.435889894	2.294157339	1
	0.99	0.9801	0.14106736	7.08881205	1
	0.999	0.998001	0.044710178	22.36627204	1
	0.9999	0.99980001	0.014141782	70.71244595	1
	0.99999	0.99998	0.004472125	223.6073568	1

Changes are very, very, very small until you get near the speed of light.

You tube: Theory of Relativity

http://www.youtube.com/watch?v=AZ6N85lNgHY&feature=PlayList&p=50193D62F125C243&index=0&playnext=1

Einstein's Big Idea

http://www.youtube.com/watch?v=V7vpw4AH8QQ&feature=PlayList&p=50193D62F125C243&index=1

Time Travel Is Possible

http://www.youtube.com/watch?v=X02WMNoHSm8&NR=1