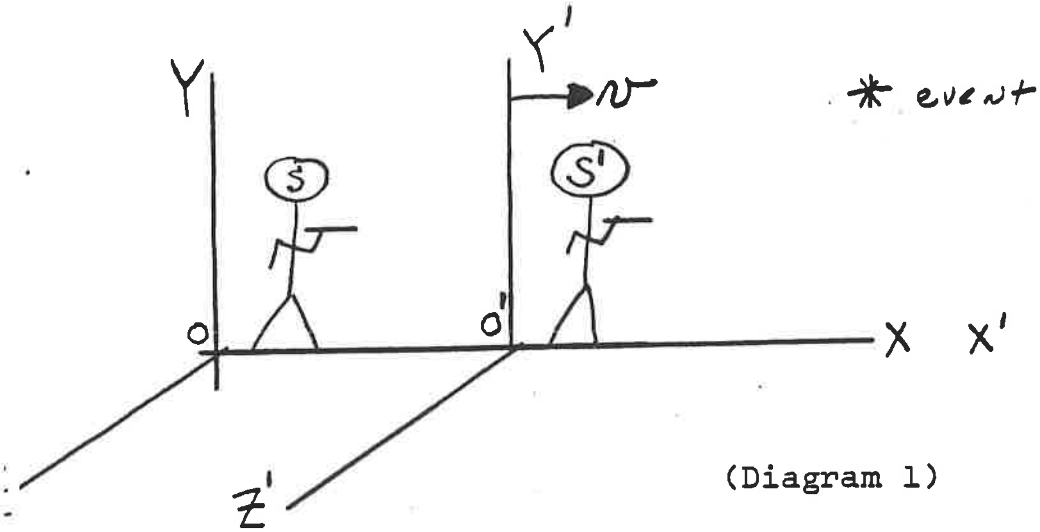
Special Relativity

<u>Introduction to Special Relativity using minimal mathematics</u>
(Relativity for retards)



(Diagram 1)

<u>Newtonian Relativity</u>- Two observers, S and S' are in reference frames x,y,o, and x',y',o' respectively. They have identical clocks which are synchronized at t=0 when the origins coincide.

S and S' have identical rulers.

Reference frame x',y',o' is moving to the right with respect to x,y,o at constant velocity v in the common x direction.

The event (which occurs at a specific location and time) is observed by both S and S'

S gives the coordinates of the event (X,Y,Z,t)

S' gives the coordinates of the event as (X',Y',Z',t')

The problem is to arrive at TRANSFORMS, which are equations which will relate coordinates in one reference frame to those in the other.

The following are called <u>Gallilean Transforms</u>, and seem to be self-evident from the diagram and discussion above:

$$X' = X - vt \qquad 1.1$$
$$Y' = Y \qquad 1.2$$
$$Z' = Z \qquad 1.3$$
$$t' = t \qquad 1.4$$

Also, if an object is MOVING, the observers will "see" different velocities $\dot{X}$ and $\dot{X}'$. The following relationship is obtained by simply taking the time derivitive of the first equation above:

$$\dot{X}' = \dot{X} - v \qquad 1.5$$

THEORY OF SPECIAL RELATIVITY

Postulates:

1. The laws of Physics are invariant in all inertial systems. (The mathematical form remains the same) (All inertial frames of reference are equivalent)
2. The speed of light in a vacuum is constant, independent of the inertial system, source, observer.

Referring back to diagram 1, if the two observers have coordinates

S says (X,Y,Z,t)
S' says (X',Y',Z',t') and we are looking for a mathematical relationship between these two reference frames. (Looking for transforms)

It seems reasonable to assume the transforms must be linear, otherwise a single event would appear as 2 events to the other observer.

Proposed transforms:

$$X' = A(X-vt) \qquad 2.1$$

$$Y' = Y \qquad 2.2$$

$$Z' = Z \qquad 2.3$$

$$t' = B(t-DX) \qquad 2.4$$

[where A,B,D are to be determined mathematically].

Let us start a spherical light pulse at the origin at the moment when the origins coincide (at t=0)

S says $X^2 + Y^2 + Z^2 = c^2t^2$

S' says $X'^2 + Y'^2 + Z'^2 = c^2t'^2$

Therefore:
$$X^2+Y^2+Z^2-c^2t^2 = 0 = X'^2+Y'^2+Z'^2-c^2t'^2$$

Substituting in 2.2 and 2.3 simplifies the above to

$$X^2-c^2t^2 = X'^2-c^2t'^2$$

Substitute in 2.1 and 2.4 to generate the following:

$$X^2 - c^2t^2 = A^2(X^2 - 2vXt + v^2t^2) - c^2B^2(t^2 - 2DXt + D^2X^2) \qquad 3$$

Now comes a little trick: since X and t are independent variables, the coefficients on each side of the equation must be equal:

(for X^2) $\qquad 1 = A^2 - c^2B^2D^2 \qquad$ 4.1

(for t^2) $\qquad -c^2 = A^2v^2 - c^2B^2 \qquad$ 4.2

(for Xt) $\qquad 0 = -2A^2v + 2c^2B^2D \qquad$ 4.3

Although equation 3 above appears somewhat intractable, what follows is simple (?) algebra, nothing more. If you read the following slowly and carefully a few kilotimes, it will seem very easy.

First solve 4.3 for A

$$A = \sqrt{c^2B^2D/v} \qquad 5$$

then substitute into 4.1

$$1 = c^2B^2D[(1/v) - D] \qquad 6.1$$

and also into 4.2

$$-c^2 = \frac{c^2B^2Dv^2}{v} - c^2B^2 \qquad 6.2$$

divide 6.2 by $-c^2$ to make it more beautiful

$$1 = B^2(1 - Dv) \qquad 6.3$$

multiply 6.1 by v

$$v = c^2B^2D(1 - Dv) \qquad 6.4$$

now divide 6.4 by 6.3 --c'mon, you can do it.

$$v = c^2D \quad \text{or} \quad D = v/c^2 \qquad 7.1$$

If you stop and look now, you will realize that you now have one of the unknown factors (D) in terms of measurables (v,c) [applause].

Substitute 7.1 into 6.3 to get

$$B = \sqrt{\frac{1}{1 - \frac{v^2}{c^2}}} \qquad 7.2$$

(sigh of relief)

Finally we substitute 7.1 into 5 to find A = B, then from 7.2 we get

$$A = \sqrt{\frac{1}{1 - \frac{v^2}{c^2}}} \qquad 7.3$$

We now have determined the functions A,B,D! [Oh joy, o rapture].

It only remains to substitute equation set 7 into equation set 2 to arrive at the transforms we want:

$$X' = \frac{X - vt}{\sqrt{1-(v^2/c^2)}} \qquad 8.1$$

$$Y' = Y \qquad 8.2$$

$$Z' = Z \qquad 8.3$$

$$t' = \frac{t - (v/c^2)\,X}{\sqrt{1 -(v^2/c^2)}} \qquad 8.4$$

Equation set 8 is often referred to as the LORENTZ TRANSFORMS.

It should be noted that when v is small compared to c, that equation set 8 reduces to equation set 1. This is necessary to preserve the validity of Newtonian Physics at low velocities.